EXAMPLE 2 ➤ The equation of the pre-image function in Figure 1-3e is $f(x) = \sqrt{1 - x^2}$. Confirm on your grapher that $g(x) = 2 + f(x - 4)$ is the transformed image function by

a. Direct substitution into the equation

b. Using the grapher's built-in variables feature

SOLUTION

a. $g(x) = 2 + \sqrt{1 - (x - 4)^2}$ Substitute $x - 4$ for the argument. Add 2 to the expression.

Enter: $f_1(x) = \sqrt{1 - x^2}$

$f_2(x) = 2 + \sqrt{1 - (x - 4)^2}$

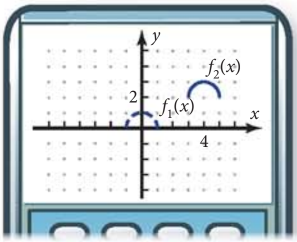

Figure 1-3f

The graph in Figure 1-3f shows an x-translation of 4 units and a y-translation of 2 units.

b. Enter: $f_3(x) = 2 + f_1(x - 4)$

The graph is the same as that for $f_2(x)$ in Figure 1-3f.

Again you may ask, "Why do you subtract an x-translation and add a y-translation?" The answer again lies in associating the y-translation with the y-variable. You actually subtract *both* translations:

$y = 2 + f(x - 4)$

$y - 2 = f(x - 4)$ Subtract 2 from both sides.

The reason for writing the transformed equation with y by itself is to make it easier to calculate the dependent variable, either by pencil and paper or on your grapher.

This box summarizes the dilations and translations of a function and its graph.

PROPERTY: Dilations and Translations

The function g given by

$$\frac{1}{a} \cdot g(x) = f\left(\frac{1}{b}x\right) \quad \text{or, equivalently,} \quad g(x) = a \cdot f\left(\frac{1}{b}x\right)$$

represents a **dilation** by a factor of a in the y-direction and by a factor of b in the x-direction.

The function h given by

$$h(x) - c = f(x - d) \quad \text{or, equivalently,} \quad h(x) = c + f(x - d)$$

represents a **translation** by c units in the y-direction and by d units in the x-direction.

Note: If the function is only dilated, the x-dilation is the number you can substitute for x to make the argument equal 1. If the function is only translated, the x-translation is the number to substitute for x to make the argument equal zero.